NUMERICAL STUDY OF POWER GENERATION FROM AN OSCILLATING AIRFOIL
NACA-4412

M. M. OUESLATI1; A. W. DAHMOUNI1; M. BEN SALAH2
C. KERKENI1; S. BEN NASRALAH3

1. Laboratoire de Maîtrise de l'Energie Eolienne et de Valorisation Energétique des DÉchets
Centre de Recherches et des Technologies de l'Energie, Technopole Borj Cedria Rue Touristique Soliman, BP 95, 2050 Hammam Lif, Tunisie.
2. Laboratoire des Procédées Thermiques, Centre de Recherches et des Technologies de l'Energie, Technopole Borj Cedria Rue Touristique Soliman, BP 95, 2050 Hammam Lif, Tunisie
3. Laboratoire d'Etudes des Systèmes Thermiques et Energétiques, École Nationale des Ingénieurs de Monastir, Rue Ibn El Jazzar, 5000 Monastir, Tunisie
E-mail: mehdi.oueslati@crten.rnrt.tn

Abstract:
The original incentive for developing the software package was the identification of the unsteady wake structure generation, and the prediction of aerodynamic performances of an airfoil. In fact, one of the key features of Laplace’s Equation is the property that allows the equation governing the flow field to be converted from a 3D problem throughout the field to a 2D problem for finding the potential on the surface.
In this work, we will interest on power generation from an oscillating airfoil. The generator consists of an airfoil which oscillates in pure plunge mode. Two-dimensional inviscid flow code is used to predict the oscillatory flow field and the aerodynamic propriety of the oscillating airfoil.

Key words: Panel method, wind turbine airfoils, incompressible potential flow, wake structure.

1. Introduction
Knowledge of wind power technology has increased over the years. Lanchester and Betz were the first to predict the maximum power output of an ideal wind turbine. The major break-through was achieved by Glauert who formulated the Blade Element Momentum (BEM) method in 1935, [Ivanell (2009)].
Almost design codes of today are still based on the Blade Element Momentum method. However, this method needs knowledge of the blade aerodynamic which depends on the airfoil nature and his intrinsic characteristic. Therefore, the aerodynamic research is today shifting toward a more fundamental approach since the basic aerodynamic mechanisms are not fully understood and the importance of accurate design models increases as the turbines are becoming larger.
The case of 2D plunging motion has been considered by many researchers to provide insight into thrust generation and propulsive efficiency.
[Tuncer and Platzer (1996)] computed the thrust force and propulsive efficiency using a Navier-Stokes (NS) code for the flow past a rigid NACA0012 airfoil undergoing pure plunging motion. The Reynolds number was 3 \times 10^6. The value of k was varied from 0.2 < k < 3 and h was varied from 0.1 < h < 0.4 to find the optimal thrust and propulsive efficiency. They found that for a single plunging airfoil, maximum efficiency as high as 0.72 can be achieved for k = 0.2 and h = 0.4 but with a very low coefficient of thrust of 0.01. They also investigated the flapping/stationary airfoil combination in tandem configuration and found that if a stationary airfoil is placed downstream of the plunging airfoil separated by two chord lengths, more than 40% gain can be achieved in efficiency and 33% in thrust coefficient at k = 0.75 and h = 0.2.
[Jones and Platzer (1997)] reported their computational results using a 2D incompressible unsteady panel method (UPM) code for flow over different airfoil sections undergoing pure plunging motion and found that
varying the thickness of the airfoil has a negligible effect on thrust generation and propulsive efficiency in the frequency and amplitude range considered, \( k = 0.01 - 10 \) and \( h = 0.1 - 0.4 \).

Panel methods were initially developed as lower order methods for incompressible and subsonic flows. The first successful panel method for supersonic flow became available in the mid-1960s developed by Woodward-Carmichael. Hess and Smith developed together the Hess-Smith code in 1962 based on flat constant source Panels. Woodward-Carmichael has used into the series of computer programs known as USSAERO in 1973. Many other numerical programs such as SOUSSA, PAN AIR, VSAERO and QUADPAN are developed with different boundary condition configurations, and different numerical techniques of resolution, Erickson (1990).

In the recently work, this method was used to predict wind turbine performance. This method was used to predict performance of the NACA 63(2)215 and the FFA-W3-241 airfoils by [Kamoun et al (2005)]. The results show a good agreement with experimental data. The method has used also to calculate the unsteady loading and radiate noise from airfoils in incompressible turbulent flow by [Glegg and Devenport (2010)]. This method is also used in the design of wind turbine blades in order to increase the energy produced by the aero-generators. A related method has used that imposes circulation instead of the blade load in the work of [Kamoun et al (2006)]. The current design method imposes indirectly the circulation by prescribing the pressure difference between both sides of the blades and hence the lift. Furthermore, the method has used in the direct analysis of fast panel method for blade cascades by [Henriques et al (2009)]. The approach starts by assuming an initial geometry and calculates the flow field caused by it. The differences between the calculated flow field and the desired one are used to modify the original geometry. In this paper we will resume the method used for steady and unsteady cases and some validations of the code.

### 2. Theoretical Method

The basic idea of the Panel method is to:

- Discretize the body in terms of a singularity distribution on the body surface.
- Satisfy the necessary boundary conditions.
- Find the resulting distribution of singularity on the surface thereby obtain fluid dynamic properties of the flow.

The body geometry is represented in terms of smaller subunits called panels, hence the name “panel” method. Each panel is constructed to have some kind of singularity distribution. The singularities used can be sources, doublets, or vortices. Depending on the accuracy, computational speed and other factors one can use constant, linear, parabolic, or even higher orders of distribution of the singularity on each panel. The number of panels that represent the body can also be varied.

The actual singularity distribution is initially unknown, but by enforcing the boundary conditions on the body, it is possible to solve for them. The boundary conditions can be represented in terms of the velocity field, called the Neumann condition, or in terms of the potential inside the body, called the Dirichlet condition.

The formulation of the panel method consists in the resolution of Laplace’s equation Eq. (1) through the superposition of simpler solutions of elementary flows distributed throughout the body. This characteristic makes the method fast, because it is not necessary the discretization of all flow domains. The Laplace equation is written as:

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0
\]  

The total potential can be written us follow:

\[
\phi = \phi_\infty + \phi_3 + \phi_Y + \phi_W
\]  

### 2.1 Steady case:

The goal is to calculate the lift coefficient and the pressure distribution on the airfoil. The steps of calculation are presented here and the detail of the mathematical model is discussed in the paper written by Oueslati et al (2011).
3.1 Unsteady case and wake evolution:

In the extension of Hess and Smith Panel method to unsteady flows, the airfoil surface is again represented by singularity distributions of source strength \( q_j \) (1...N) and vorticity \( \Gamma \) leading to (N+1) unknowns us in steady flow. For unsteady flows, however, these unknowns are time dependent. Since the total circulation in the flow field must be preserved according to Helmholtz theorem, any changes in the circulation \( \Gamma \) on the airfoil surface must be balanced by an equal and opposite change in the vorticity in the wake which causes vortex shedding at the trailing edge of the airfoil. In the unsteady panel method, the vortex shedding is represented by an additional wake element, “the shed vorticity panel” attached to the trailing edge with uniform vorticity distribution \( (\Gamma_{sh})_k \), see figure 1.

If the length of the shed vorticity panel is denoted by \( \Delta_k \) and its inclination angle to the x-axis by \( \theta_k \) and the overall circulation of the airfoil surface at time step \( t_k \) by \( \Gamma_k \), with \( l \) denoting the perimeter of the airfoil, then the Helmholtz theorem can be expressed as:

\[
\Gamma_k + \Delta_k (\Gamma_{sh})_k = \Gamma_k \quad \text{or} \quad \Delta_k = \Gamma_k - \Gamma_k = \frac{l}{l} (\Gamma_{sh} - \Gamma_k)
\]

(3)

Here \( \Gamma_{sh} \) and \( \Gamma_k \) represent the overall circulation on the airfoil surface and the vorticity strength, respectively, already determined at time step \( t_{k-1} \).

From time step \( t_k \) to \( t_{k-1} \), we assume that the shed vorticity panel is detached from the trailing edge and convects downstream as a concentrated free vortex, with circulation \( \Delta_k (\Gamma_{sh})_k \). The convection velocity of the free vortex is given by the local flow velocity at the center of the vortex. At the same time, a new shed vorticity panel is formed and the vortex shedding process is repeated. As a result of this continuous vortex shedding, a string of concentrated core vortices is formed in the wake behind the airfoil shown in figure 1.

Two additional relations are required and can be obtained from the assumptions recommended by Basu and Hancock:

1. The shed vorticity panel is oriented along the direction of the local flow velocity at the panel midpoint.
2. The length of the shed vorticity panel is equal to the product of the local flow velocity at the panel midpoint and the size of the time-step.

With these assumptions, the two additional relations can be written as:

\[
\begin{align*}
r \tan \theta_k &= \left( \frac{v_x}{u_x} \right)_k \\
\text{And} \quad \Delta_k &= (t_k - t_{k-1}) \left[ (u_x)_k \right]^2 + (v_x)_k \right]^3
\end{align*}
\]

(4)

(5)

where \( (u_x)_k \) and \( (v_x)_k \) are the flow velocity components at the panel midpoint in the x- and y-directions, respectively.

The flow tangency conditions remain the same as those for steady flows given by:

\[
\left[ (V^2)_{y,i} \right] = 0 \quad i=1,2,...,N
\]

(6)

However, the Kutta condition must now include the rate of change of velocity potential. According to the unsteady Bernoulli’s equation, the Kutta condition can be expressed by:

\[
\left[ (V^2)_{y,i} \right] = 2 \left[ \frac{\partial (\phi_x - \phi_y)}{\partial x} \right]_i = 2 \left[ \frac{\partial \phi_x}{\partial x} \right]_i
\]

(7)

Using a backward finite-difference approximation for the rate of change of total circulation on the airfoil surface, can be written as:
The Kelvin condition must be added to the unsteady model, it is given as
\[ \frac{\partial \Gamma}{\partial t} = 0 \]  
This means that if the circulation of the airfoil changes then circulation of opposite sign must be added to the 
flow domain to counteract the change.

3. Power Generation and Efficiency

To generate power, two significant changes must be made by the airfoil. First, the airfoil is oscillating in pitch 
and second in plunge according to:
\[ \tilde{h} = H \sin(2\pi f t) \tilde{j} \] 
\[ \tilde{a} = (\Lambda + \alpha_n \sin(2\pi f t)) \tilde{k} \]  
we must modify the value of the velocity at infinity in the moving reference frame. Thus:
\[ \tilde{V}_{\text{stream}} = \tilde{V}_a - \tilde{V}_{\text{airfoil}} = \tilde{V}_a - (\tilde{h} + \tilde{\alpha} \times \tilde{r}_{\text{ref}}) \]  
where
\[ \tilde{h} = 2\pi f H \cos(2\pi f t - \delta) \tilde{j} \] 
and
\[ \tilde{\alpha} = 2\pi f \alpha_n \cos(2\pi f t) \tilde{k} \]  
Since the intent of this paper is to calculate the power generated by an unsteady airfoil this section follows the 
method of McKinney for calculating power:
\[ P = \tilde{h} [N \cos(\alpha) + (T_\delta - D) \sin(\alpha)] + \tilde{\alpha} M \] 

The leading edge suction was found to be negligible for the wingmill tested and the only drag force in our 
coordinate system is orthogonal to the vertical velocity, therefore this equation reduces to:
\[ P = \tilde{h} N \cos(\alpha) + \tilde{\alpha} M = \tilde{h} L + \tilde{\alpha} M \] 
where N and M are found from summing the forces induced by the pressure distribution. Note that since P is an 
instantaneous power, it is not a terribly useful quantity unless it is averaged over an integral number of cycles.

The efficiency may be found by comparing the power produced to that of an ‘ideal’ wingmill which is given by:
\[ \bar{P}_{\text{ideal}} = \frac{16}{27} \frac{1}{2} \bar{\rho} V^2 \alpha \] 
\[ \bar{A} = 2b(h_{1e})_{\text{max}} \]  
Thus:
\[ \eta = \frac{\bar{P}}{\bar{P}_{\text{ideal}}} \]  

4. Results and discussions

4.1 Unsteady wake validation

In the unsteady case we will interest, in the first hand, to the wake structure behind an airfoil in the plunging 
movement. The plunging is defined as a vertical flapping at right angles to the direction of motion. The NACA 
0012 airfoil with chord length c performs a sinusoidal plunging. The position of the airfoil is
\[ y(t) = y_0 \sin(\omega t) \]
Where $t$ is physical time, $\omega$ and $\gamma_0$ are the angular frequency and the amplitude of plunging motion, respectively.

The nondimensional plunge amplitude is defined as $h = \frac{2c}{c}$, the reduced frequency is defined as $k = \frac{\omega c}{2U_\infty}$ and the instantaneous nondimensional plunging velocity is:

$$\frac{\dot{y}}{U_\infty} = 2hk \cos(\omega t)$$

(21)

The maximum effective angle of attack is $\alpha_{\text{max}} = \tan^{-1}(2hk)$ which occurs at $y = 0$. The time averaged angle of attack over a plunging cycle is zero. The characteristics of the motion used here in the computation test are: $c = 1$, $y_0 = 0.019$, $dt = 0.0057$, $\omega = 26.829$, and $U_\infty = 1\text{ms}^{-1}$.

Fig. 2. Comparison between our numerical solution with numerical results of plunging NACA 0012 of Katz & Weihs (1978)

We obtain the result in the figure below. We can observe clearly the oscillation of the wake in function of the reduced frequency. The numerical result shows a good agreement compared with experimental visualization of Katz & Weihs (1978).

4.2 Effect of heaving frequency oscillation on wake pattern of NACA 4412

To study the effect of heaving frequency oscillation on wake pattern of NACA 4412, three cases have been made for a constant plunge amplitude equal to $h = 0.08$ and reduced frequency equal to 5, 10 and 15 respectively.

Fig. 3. Wake pattern of NACA 4412 for reduced frequency $k = 5, 10, 15$ respectively.
We can observe that with increasing frequency the vortex becomes closer and the downstream tilt growth with the reduced frequency. We observe also, that is for pure plunging motion the wakes of all value of are $k = \frac{U_m}{2U}$ thrust-producing with downstream tilted vortex pair.

5. Conclusion

The Panel method is used to develop a steady an unsteady numerical code to predict airfoil performances. The UPM code predicts thrust-producing wakes for all values of $k$ with some variation in structure dependent on $k.h$ but also varying with $k$ independently.

The UPM code cannot account for viscous drag from the airfoil since it uses an inviscid formulation, however in reality there would be additional effects due to this drag component.

6. References